

# A MECHANISM FOR ELECTROMAGNETIC TRAPPING OF EXTENDED OBJECTS

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**PACS :** 52.55.-s 45.40.-f

**Keywords :** Axisymmetric lift field Octupolar trapping field Rotating saddle trap

In this work a mechanism is proposed for levitation and stable confinement of a heavy rotating body using electromagnetic interactions alone, with no position-dependent control. Support of the weight and vertical confinement are achieved through an axisymmetric magnetic field while lateral confinement is accomplished by a rotating octupolar field via the Brouwer saddle mechanism. The orientational degrees of freedom are stabilized through gyroscopic action. The design features multiple variable parameters, thus allowing considerable flexibility for scaling the system size and maximizing the basin of attraction of the stable state.

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**E**arnshaw's theorem states that that no configuration of charges or currents (or hypothetical magnetic monopoles) can be stably confined in space by any time-independent configuration of charges or currents in air or media with positive permittivity and permeability [1,2]. This raises difficulties in the design of electromagnetic traps for ions, atoms or, in particular, rigid bodies. Perhaps the first instance of magnetic levitation was in 1939 when the German physicist BRAUNBECK levitated some diamagnetic graphite particles above a strong electromagnet. Sixty years later, SIR ANDRE GEIM, SIMON and co-workers performed a highly innovative variant of this using a 16 Tesla magnetic field and a live frog as the levitator [3-5]. When high  $T_c$  superconductivity was discovered [6], it fuelled dreams of the ultimate levitator based on this principle [7], but as we know, such a concept has yet to fructify.

Microscopic particle traps usually use time dependence to work round Earnshaw's theorem. Paul trap [8-11] consists of an electric field which is statically stable in two directions but unstable in the third, and then oscillates rapidly to stabilize the troublesome direction via the Kapitza pendulum mechanism. In Penning trap, the electric and magnetic fields are static but the electrons themselves move through the interior of the trap [12,13]. The ingenuity of the design ensures that the electron is forever unstable to a perturbation along its path (there must be an unstable direction as per Earnshaw) but the path is closed, and stable to deviations perpendicular to itself. These two concepts ensured that the problem of electromagnetic trapping of microscopic particles was essentially solved. Although scope for improvement in design always remains, the basics are by now well established.

Electromagnetic trapping of rigid bodies presents a different story. The existing levitators use brute force to achieve confinement – they mount an array of electromagnets all connected to variable sources, continuously sense the position of the object and instantaneously excite the right set of electromagnets to counteract its immediate destabilization tendency [14]. Every dynamical system, however unstable intrinsically, can always be stabilized through a sufficiently complex control arrangement, and this is the philosophy employed by these devices. This method of bypassing Earnshaw's theorem is inelegant as well as impractical, as evident from the fact that maglev trains today carry passengers on 59 km of track worldwide.

The only design till date which achieves control-free magnetic levitation and stabilization of a macroscopic object is the toy called “Levitron”, invented by HARRIGAN and HONES. This is a magnetic dipole in the shape of a top which floats above a magnetic block. Its action was first explained by SIR MICHAEL BERRY [15], and I will hereafter refer to the Levitron as the HHB confinement mechanism. Amazing as this mechanism is, with a dc

magnetic field achieving trapping and with the subtle underlying physics connecting to various aspects of quantum theory [16-18], it has not proved practically realistic due to the light weight of the top, the smallness of the basin of attraction of the stable state and the absence of external parameters which can be tuned to scale the system size or enhance its stability. Twenty years after its invention, the HHB mechanism still powers only a fascinating toy and nothing of greater import.

In this Letter I will propose a novel design for an electromagnetic rigid body trap. Like the HHB mechanism, it uses no active control but the manner of bypassing Earnshaw's theorem is closer in spirit to that of Paul trap. This enables the system size to be scaled as per the application requirements and allows for the introduction of several design parameters which can be optimized for maximal stability. The design of this trap is shown below.

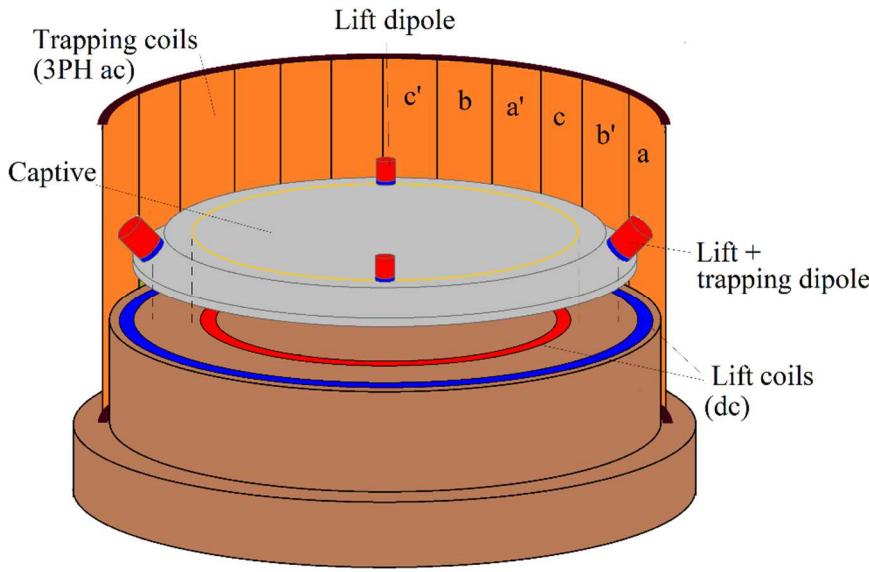


Figure 1 : Schematic representation of the trap. The "captive" is the rigid body which is to be confined; in general it spins fast about its symmetry axis. For the magnetic dipoles, red denotes North and blue South. The lift dipoles are mounted vertically, pointing upwards. The two inclined dipoles can be decomposed into a vertical lift component and a horizontal outward-pointing trapping component. The lift coil with red face carries current counter-clockwise (CCW) while that with blue face carries clockwise (CW) when viewed from the top. The three-phase octupolar winding encloses the apparatus completely; half of it has been excised to reveal what lies inside. a,b,c here denote the phase names and not the Eulerian basis of (3).

The electromagnetic force and torque on each dipole can be written as

$$\mathbf{F} = \sum_{\text{all dipoles } n} (\mathbf{m}_n \cdot \nabla) \mathbf{B}_n , \quad (1a)$$

$$\mathbf{T} = \sum_{\text{all dipoles } n} (\mathbf{r}_n \times \mathbf{F}_n + \mathbf{m}_n \times \mathbf{B}_n) , \quad (1b)$$

where the summations run over the lift as well as the trapping dipoles and  $\mathbf{B}_n$  denotes the magnetic field at the location of dipole  $n$  arising from the lift as well as the trapping coils. Note that in (1b),  $\mathbf{r}_n$  denotes the vector from the captive centre of mass (CM) to the  $n^{\text{th}}$  dipole. The translational equations of motion are straightforward :

$$\ddot{x}_{CM} = F_x / M , \quad (2a)$$

$$\ddot{y}_{CM} = F_y / M , \quad (2b)$$

$$\ddot{z}_{CM} = F_z / M - g , \quad (2c)$$

where  $M$  is the captive mass. For the rotational equations we use Euler angles in the Euler or gyroscope convention [19]. Letting the captive be symmetric with axial moment of inertia  $I_c$  and perpendicular moments equal to  $I$ , we define the  $a,b,c$  basis as one which shares the precession and nutation of the captive but not its spin. Here  $a$  is the nutation axis and  $c$  the symmetry axis. Evaluating the material derivative of angular momentum in this basis yields

$$I\ddot{\theta} + I_c(\dot{\psi} + \dot{\phi}\cos\theta)\dot{\phi}\sin\theta - I\dot{\phi}^2\cos\theta\sin\theta = T_a , \quad (3a)$$

$$I\ddot{\phi}\sin\theta + (2I - I_c)\dot{\phi}\dot{\theta}\cos\theta - I_c\dot{\theta}\dot{\psi} = T_b , \quad (3b)$$

$$I_c(\ddot{\psi} + \dot{\phi}\cos\theta + \dot{\phi}\dot{\theta}\sin\theta) = T_c . \quad (3c)$$

These are the equations of motion of the captive.

Before attempting a mathematical treatment of these equations, we will spend some time to argue on physical grounds that the trap based on this design will really work. A physical argument has the advantage of not being reliant on approximations, which are essential to get meaningful quantitative results for such a complicated system. We start from an analysis of the Brouwer saddle [20-23], which forms the core of the design. The basic equation of this saddle can be written as

$$\ddot{x} = 2A(-x\cos\omega t + y\sin\omega t) , \quad (4a)$$

$$\ddot{y} = 2A(x \sin \omega t + y \cos \omega t) , \quad (4b)$$

where  $x$  and  $y$  are the displacements of the particle,  $A$  the strength of the saddle and  $\omega$  is twice of its rotation frequency. Following LANDAU's treatment of the Kapitza pendulum we do a direct partition of motion into slow and fast, writing  $x=x_s+x_f$  and  $y=y_s+y_f$  where the 's' components vary slowly and the 'f' components vary rapidly, oscillating at frequency  $\omega$ ; solving for the fast components and averaging over them gives the slow equations

$$\ddot{x}_s = -\frac{4A}{\omega^2} x_s , \quad (5a)$$

$$\ddot{y}_s = -\frac{4A}{\omega^2} y_s , \quad (5b)$$

which clearly describes a spring in both directions. Numerical work shows clear-cut stability zones in the  $(A, \omega)$  parameter plane; the quantitative values (5) have some scope for improvement. Much more detailed and accurate analysis can be found in the References cited earlier.

Now we see how this concept gets applied in this trap, to overcome the instability arising from the lift coils. These coils support the weight of the captive by generating a field such that  $\partial B_z / \partial z > 0$ . The second derivative  $\partial^2 B_z / \partial z^2$  is positive at small  $z$ , has a zero at some finite value of  $z$ , and is negative thereafter. If we choose the default operating point (the "cruising altitude") to be in the latter region, then the system will automatically be stable in  $z$ . By Earnshaw's theorem and circular symmetry, it must be unstable in  $x, y$ . To counter this instability we introduce the trapping field and dipoles; since they are all in the plane, they do not tamper with the  $z$ -stability. The trapping current can be written to good approximation as a surface current (Amps per metre)  $\mathbf{K} = (K_0 \cos n(\gamma - \zeta)) \hat{z}$ , where  $2n$  is the polarity of the trapping coils,  $\gamma$  the azimuthal angle and  $\zeta$  a phase which is in general a function of time. This creates a field which lies in the  $x-y$  plane and possesses a spatial gradient whose strength increases with  $n$ . In this Letter we take the case  $n=4$  (quadrupolar trapping coil), which produces a favourable interaction with two trapping dipoles on the captive. A schematic of this is shown in Fig. 2.

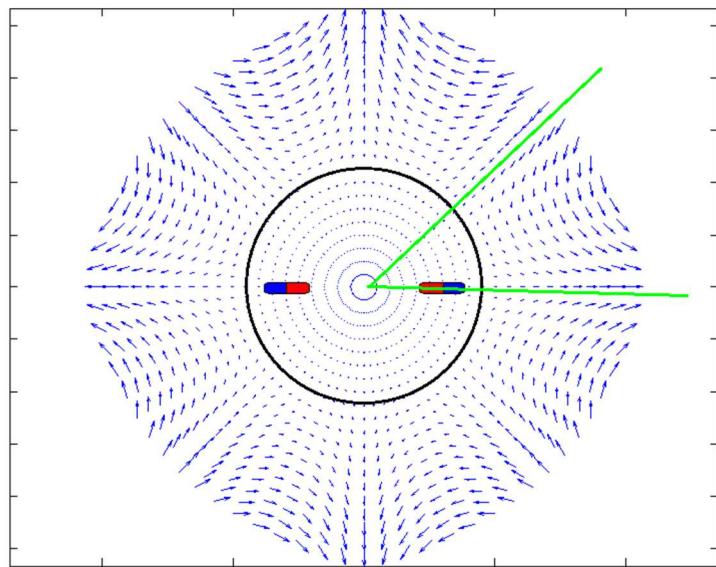


Figure 2 : The octupolar magnetic field with the captive inside. The green line to the right is the  $x$ -axis and the other one makes a  $45^\circ$  angle to it. Its significance is explained in the text.

In the configuration shown in the Figure, the trapping field points dead outwards along the  $x$ -axis which is how the dipoles are aligned. When the captive is at the centre of the trapping field, the force on the right dipole is leftwards, that on the left dipole is rightwards, and their resultant is zero. If the captive is displaced slightly along the  $x$ -axis to the right, then the force on the right dipole becomes stronger still while that on the left one becomes weaker, creating a resultant opposite to the displacement. Consistent with Earnshaw's theorem, if the captive is displaced perpendicular to this line, then the resultant force will augment the displacement, creating the expected saddle behaviour. Now consider the case where the captive has rotated and is aligned with the  $45^\circ$  line. This time, the trapping field points dead inwards along this line, so the situation is the same as the previous case except for the signs. The captive augments a displacement along the line while opposing one perpendicular to the line.

This is a saddle where the axes have switched. Hence as the captive rotates through the octupolar field, it effectively sees a rotating saddle potential, which is the Brouwer saddle. This action counteracts the  $x, y$  instability arising from the lift coils, and generates overall translational stability. The rotational degrees of freedom are stabilized primarily through the gyroscopic action of the captive, which is pretty standard and needs no separate elaboration.

We now express the above arguments in quantitative terms. Due to the difficulty of analytic characterization of the magnetic field, we shall not attempt to convert (1-3) into an explicit nonlinear equation of motion. Rather, considering the reference configuration to be where the captive is coaxial with the lift and trapping coils and is at a cruising altitude of  $z_0$  above the load coils and at zero nutation, we invoke symmetry arguments and/or elementary calculation to prove that it is a fixed point of the nonlinear system, and then perform a stability analysis based on ad hoc linearization about that point. We let the perturbations be  $x_{CM}$  (deviation of captive CM from  $x=0$ , which is the

abscissa of the centre of the lift coil),  $y_{CM}$  (defined analogously),  $z_{CM}$  (deviation of captive CM from  $z=z_0$  whose value is chosen to ensure static stability in  $z$ ) and  $\theta$  (the angle of nutation of the captive).

Letting the captive radius  $r_0$  be equal to the average of those of the CW and CCW lift coils, the lift field can be written in a narrow annulus about  $(r_0, z_0)$  as

$$B_\rho = 0 - C_1(\rho - r_0) + C_2(\rho - r_0)(z - z_0) + \dots , \quad (6a)$$

$$B_z = -B_0 + C_1(z - z_0) + \frac{1}{2}C_2[(\rho - r_0)^2 - (z - z_0)^2] + \dots , \quad (6b)$$

where  $B_0$ ,  $C_1$  and  $C_2$  are positive. This expression accounts for the facts that at  $z=z_0$ ,  $B_z < 0$ ,  $\partial B_z / \partial z > 0$  as required for a lift force, and  $\partial^2 B_z / \partial z^2 < 0$  as required for static stability; at  $\rho=r_0$ ,  $B_\rho \sim 0$  as seen from analytical and numerical work, and that  $\nabla \cdot \mathbf{B} = 0$ ,  $\nabla \times \mathbf{B} = 0$ . We let  $m_1$  be the strength of the lift dipoles and  $m_2$  that of the trapping dipoles; we assume that the latter point radially inwards. The number of lift dipoles is chosen so as to cancel off certain second harmonics, while that of trapping dipoles is selected in view of considerations arising from the trapping field. By Taylor expanding the magnetic field to first order in perturbations and using (1), we can write the force as

$$F_x = \frac{C_1 m_2}{r_0} \left[ x_{CM} \left\{ 1 - \cos 2(\varphi + \psi) \right\} + y_{CM} \left\{ \frac{1}{2} \sin 4(\varphi + \psi) \right\} \right] - 2C_1 m_1 \theta \sin \varphi + C_2 m_1 x_{CM} , \quad (7a)$$

$$F_y = \frac{C_1 m_2}{r_0} \left[ x_{CM} \left\{ -\frac{1}{2} \sin 4(\varphi + \psi) \right\} + y_{CM} \left\{ 1 - \cos 2(\varphi + \psi) \right\} \right] + 2C_1 m_1 \theta \cos \varphi + C_2 m_1 y_{CM} , \quad (7b)$$

$$F_z = -2C_2 m_1 z_{CM} , \quad (7c)$$

and the torque as

$$T_x = 2r_0 m_2 \left[ -\frac{C_2}{2} x_{CM} \sin 2(\varphi + \psi) + C_2 y_{CM} \sin^2(\varphi + \psi) - 3C_1 \theta \sin \psi \sin(\varphi + \psi) \right] + 4m_1 B_0 \theta \cos \varphi + 2C_1 m_1 y_{CM} , \quad (8a)$$

$$T_y = 2r_0 m_2 \left[ C_2 x_{CM} \cos^2(\varphi + \psi) + \frac{C_2}{2} y_{CM} \sin 2(\varphi + \psi) + C_1 \theta \sin \psi \cos(\varphi + \psi) \right] - 4m_1 B_0 \theta \sin \varphi - 2C_1 m_1 x_{CM} . \quad (8b)$$

The role of  $T_z$  is to affect the rotation speed of the captive. Since that is physically constrained to remain constant on the long term, and is controlled by the user depending on the application requirement,  $T_z$  is not a quantity of interest.

We assume that the cylindrical trapping coil is infinitely long in  $z$  and carries a surface current (Amps per metre)  $\mathbf{K} = K_0 \cos 4\gamma \hat{\mathbf{z}}$  where  $\gamma$  denotes azimuthal angle in cylindrical coordinates. Inside it we can write the magnetic field in Cartesian coordinates as  $\mathbf{B} = (\mu K_0 / 2R^3) [ (x^3 - 3xy^2) \hat{\mathbf{x}} + (-3x^2y + y^3) \hat{\mathbf{y}} ]$ . Due to the rotation of the three-phase current however, this expression will in general be valid not in the  $x, y$  basis but in a new basis  $x_r, y_r$  which makes an angle  $\zeta(t)$  with the  $x, y$  basis. Thus  $\mathbf{B}$  will actually be the above expression with  $x, y$  replaced by  $x_r, y_r$ , and the conversion between the bases will be given by

$$\begin{bmatrix} x_r \\ y_r \end{bmatrix} = \begin{bmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad \begin{bmatrix} B_x \\ B_y \end{bmatrix} = \begin{bmatrix} \cos \zeta & -\sin \zeta \\ \sin \zeta & \cos \zeta \end{bmatrix} \begin{bmatrix} B_{xr} \\ B_{yr} \end{bmatrix} , \quad (9)$$

The calculation of the first and second derivatives of this can be simplified by recognizing that we require these derivatives at the default (unperturbed) location of the  $n^{\text{th}}$  lift or trapping dipole, which has the form  $r_0 \cos(\varphi + \psi + \beta_n) \hat{\mathbf{x}} + r_0 \sin(\varphi + \psi + \beta_n) \hat{\mathbf{y}}$ , where  $\beta_n$  is the angle made by the position vector from the captive CM to the dipole with a body-fixed in-plane axis. Then, to account for the rotation of the field, we can simply deduct an angle  $4\zeta$  from the arguments of these trigonometric functions and substitute these angles into the derivatives of the unrotated field. The factor of 4 arises from the fourfold symmetry of the octupolar field.

The following linearized expressions for force are obtained from a lengthy calculation :

$$F_x = \frac{6\mu K_0 r_0 m_2}{R^3} \left[ -x_{CM} \cos 2(\varphi + \psi - 4\zeta) + y_{CM} \sin 2(\varphi + \psi - 4\zeta) \right] , \quad (10a)$$

$$F_y = \frac{6\mu K_0 r_0 m_2}{R^3} \left[ x_{CM} \sin 2(\varphi + \psi - 4\zeta) + y_{CM} \cos 2(\varphi + \psi - 4\zeta) \right] , \quad (10b)$$

and  $F_z = 0$  because of the absence of  $z$ -dependence of the trapping field. This is of course the Brouwer saddle (4). This result has in fact been applied to particle traps [24,25]; here we can see a novel application to a rigid body trap. If we

average over the fast frequency scales in (7a,b), we can see on a slow level the repeller terms  $C_2 m_1$  which arise from Earnshaw (recall that (7c) shows static stability). This repeller effect can be countered by the rotating saddle. We note that if the number of trapping dipoles had been four, then (10) would have got replaced by  $F_x = F_y = 0$ , which is useless.

The torque from the trapping field is

$$T_x = \frac{3\mu K_0 r_0^3 m_2}{R^3} \theta \sin \psi [\cos(\varphi + \psi) \sin 2(\varphi + \psi - 4\zeta) - \sin(\varphi + \psi) \cos 2(\varphi + \psi - 4\zeta)] - \frac{\mu K_0 r_0^3 m_2}{R^3} \theta \sin \psi \sin 3(\varphi + \psi - 4\zeta) , \quad (11a)$$

$$T_y = \frac{3\mu K_0 r_0^3 m_2}{R^3} \theta \sin \psi [-\cos(\varphi + \psi) \cos 2(\varphi + \psi - 4\zeta) - \sin(\varphi + \psi) \sin 2(\varphi + \psi - 4\zeta)] - \frac{\mu K_0 r_0^3 m_2}{R^3} \theta \sin \psi \cos 3(\varphi + \psi - 4\zeta) , \quad (11b)$$

and once again  $T_z$  is not of interest.

Combining (8) and (11) and averaging over fast scales, we can see two types of torque terms – some about a fixed axis and some about the  $a$ -axis along which the rotor has nutated. The latter terms again comprise a dc component and a slowly oscillatory one. The dc one is

$$T_a = \theta (4m_1 B_0 - 3C_1 m_2 r_0) , \quad (12)$$

which contains a clearcut instability term and a second term where the sign of  $m_2$  determines stable or unstable. Nevertheless, if the captive spins fast enough, any instability here can be absorbed into precession via the gyroscopic effect and prevents the captive from toppling over. The fixed-axis terms and slowly oscillating  $a$ -axis terms are

$$T_x = (2C_1 m_1 - C_2 m_2 r_0) y_{CM} , \quad (13a)$$

$$T_y = -(2C_1 m_1 - C_2 m_2 r_0) x_{CM} , \quad (13b)$$

$$T_a = \theta \left[ \frac{3\mu K_0 r_0^3 m_2}{2R^3} \cos 4\zeta \right] . \quad (13c)$$

However, we recognize that the terms in (13a,b) are actually also oscillatory because  $x_{CM}$  and  $y_{CM}$  are themselves sinusoidal on account of the Brouwer saddle effect. The stability of the gyroscope system in the presence of an oscillatory torque about a fixed axis can be calculated by assuming fast spin, replacing the oscillation with periodic, impulsive kicks and then tracking the trajectory of the tip of the angular momentum vector. An easy reduction to a Poincare map shows that the system remains stable.

Simulation of the system confirms that the system indeed has regions of stable operation. For these simulations we have reformulated the problem in terms of the BRIAN TAIT or aircraft angle convention because it lacks the singularity (gimbal lock) present in the gyro convention at  $\theta=0$ . The lift and trapping magnetic fields are calculated using Biot-Savart law and the analytical octupolar field formula. The values I have chosen for these simulations are (in arbitrary units)

- **Lift coils** : Inner coil radius 1.00, Outer coil radius 1.10,  $\mu i / 4\pi$  set equal to 1
- **Trapping coils** :  $\mu K_0 / 2R^3$  set equal to 16000, Spin rate variable
- **Captive** : Radius 1.05, Mass 1000, Acceleration due to gravity 10, Axial moment of inertia 5000, Transverse moment of inertia 2500, Number of load dipoles 4, Strength of load dipoles 10, Number of trapping dipoles 2, Strength of trapping dipoles 10

The value of  $z_0$  is chosen as 0.1, where the lift force is 10000. This altitude is considerably above the threshold where  $\partial^2 B_z / \partial z^2$  changes from positive to negative (at around  $z=0.05$ ). Finally, I have added first order damping to the system to accelerate the convergence of the simulation. This damping is proportional to the velocity or angular velocity by a factor whose value is 1 for the translational modes and 0.4 for the rotational ones. We note that such damping cannot overturn the effect of a repelling potential;  $\ddot{x} + \gamma \dot{x} - x = 0$  is always unstable, however large  $\gamma$  may be. The physical explanation for this is that the particle becomes slow enough that the damping force is insignificant and then crawls out along the potential.

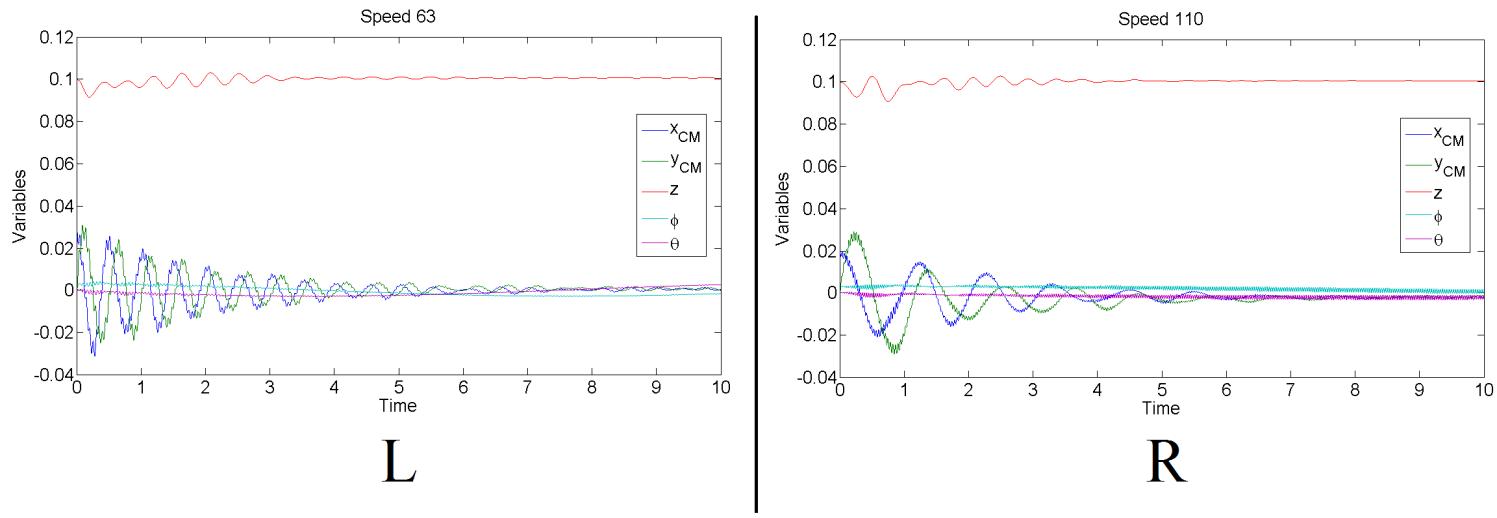


Figure 3 : Simulation of the system at two different operating points (the captive speed is labelled) with an initial perturbation in  $x_{CM}$  equal to 0.03 in the left panel and 0.02 in the right.

These preliminary simulations clearly indicate that the captive is stable and hence the proposed design of trap is effective.

We now perform further simulations to determine the regions in parameter space where the trap is stable. The parameters we have focussed on for this run are the combination  $\mu K_0 / 2R^3$  (which we rename as  $K_s$ ), the speed  $\omega$  of the captive and the slip frequency which is the difference  $\omega - 4\dot{\zeta}$ . Seeing that the stability is governed primarily by the translational modes rather than the rotational ones (except at extremely low absolute values of  $\dot{\zeta}$  which are rejected), I have run the simulations with the translational modes only to accelerate convergence and improve accuracy. For each simulation I have started the system with an initial perturbation  $x_{CM}=0.02$  and have measured whether the perturbations are growing or decaying in a time span of 5 time units. The results are below.

The first plot is for a fixed slip frequency of 50, while  $K_s$  and  $\omega$  are varied.

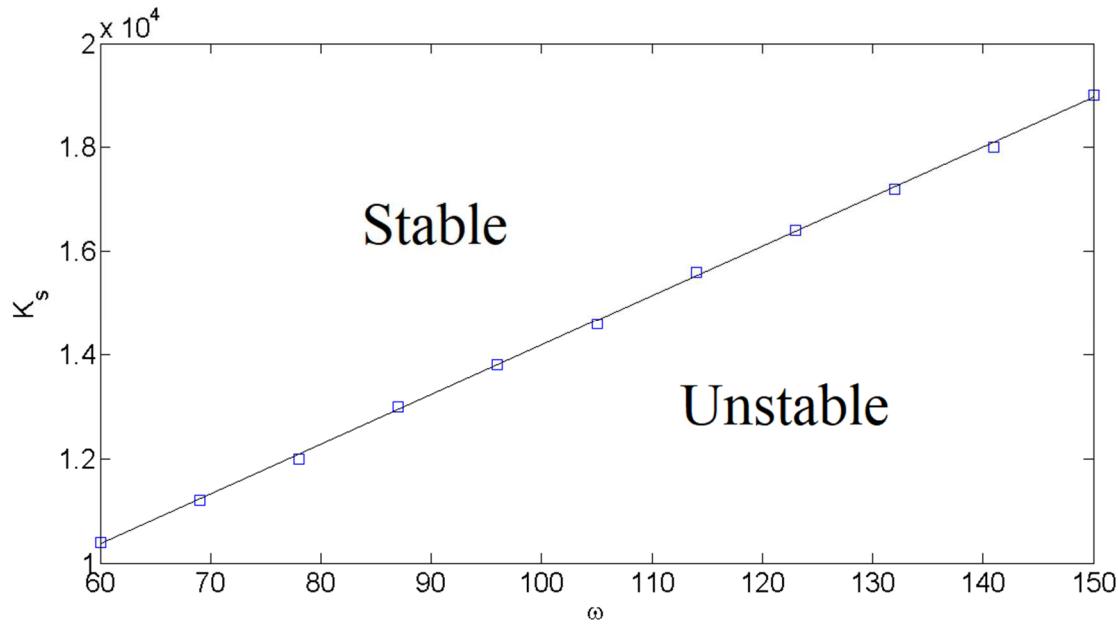


Figure 4 : Plot of system stability as  $K_s$  and  $\omega$  are varied.

There is a single transition from unstable to stable as the current is raised. We now hold  $K_s$  constant at 14000 and vary  $\omega$  and the slip frequency. This time there are two transitions – unstable to stable at a low slip and then back to unstable as the slip is raised further. Since low values of  $\dot{\zeta}$  (less than 2-3 units) cause rotational instability, I have cut off the slip at  $\omega=4$ .

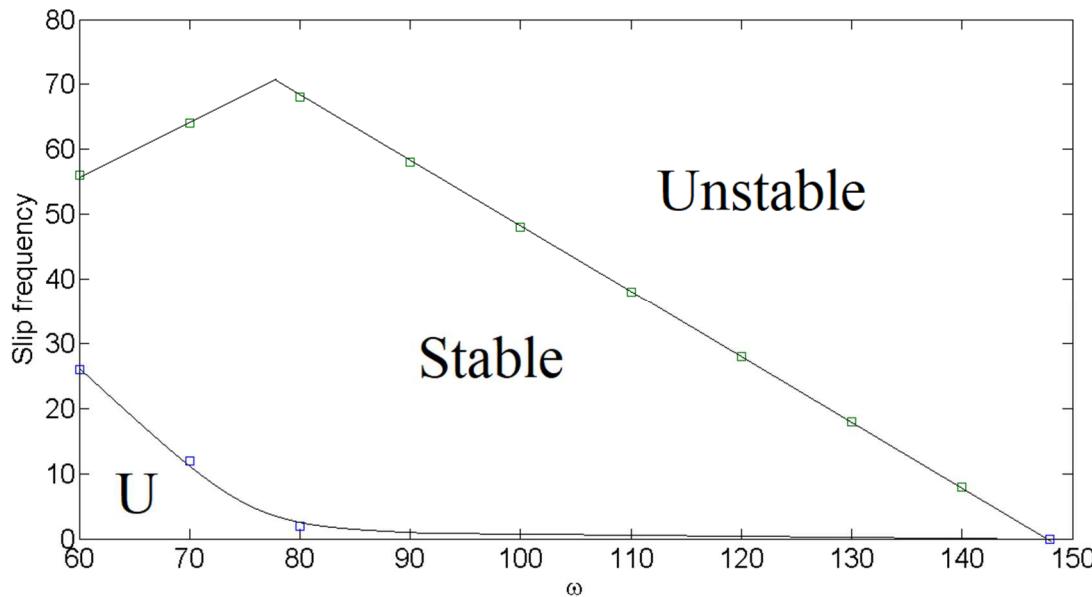


Figure 5 : Plot of system stability as  $\omega$  and slip frequency are varied. "U" denotes unstable.

Finally, I have held speed constant at 120 units and varied current with slip frequency. Once again we see a double transition, first at very low slip and then again at higher slip.

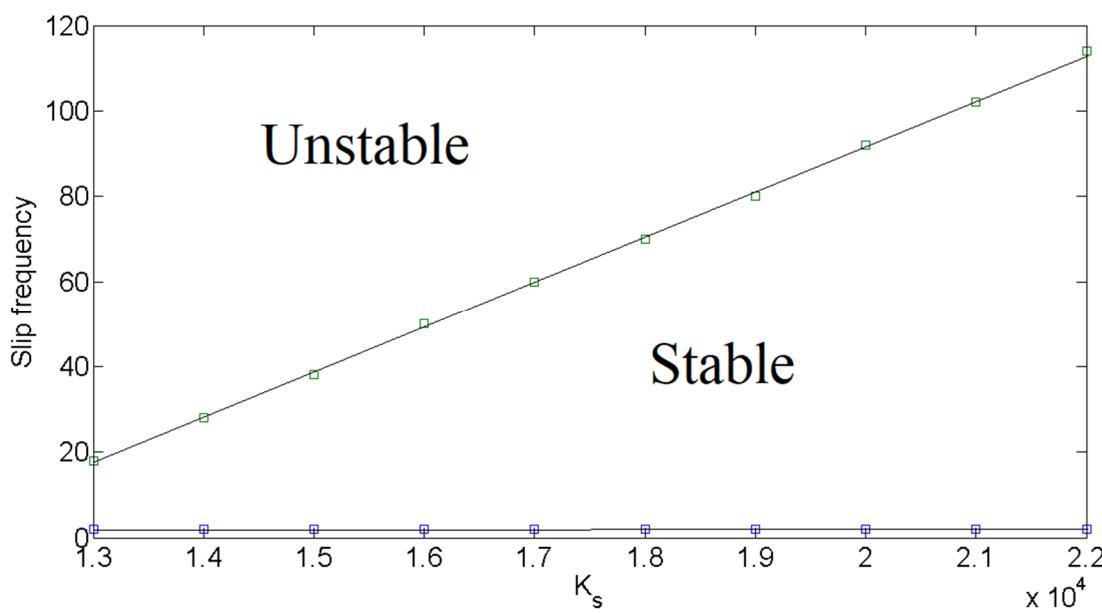


Figure 6 : Plot of system stability as  $K_s$  and slip frequency are varied.

More detailed curves can be obtained from a systematic analysis of the high order nonlinear equations governing the motion of the captive; we leave such considerations for future study.

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I am very grateful to the anonymous Reviewer for suggestion the inclusion of Figures 4-6 and providing many suggestions for improvement of quality of the manuscript.

I certify that the simulations here have been performed honestly. A legitimate demand for the code from a bonafide third party will be complied within 24 hours except under exceptional circumstances.

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